Cosmic Chemistry: Planetary Diversity

Stochastic Processes: Out of Chaos

STUDENT TEXT

NASA

When we examine the heavens there seems to be a great deal of order to the appearance and movement of the celestial bodies (stars, planets, asteroids, etc.). Since the dawn of our species, humans have speculated on how these bodies were formed and on the meaning of their movements. Most observations of natural phenomena support the contention that nature is ordered. The force that brought about this order differs depending upon the source of the explanation of how this order came to be. For most of human history, super-natural forces were credited with the imposition of order on nature. A tradition that begins with the Greek natural philosophers (circa 600 - 200 BC) and continues through contemporary science holds that change and the order of nature are the result of natural forces. What is the role of stochastic processes in a universe with such order?

STOCHASTIC PROCESSES

Stochastic is a fancy term that means the outcome of a process or event involves the occurrence of random chance events. When stochastic processes occur, there is no governing design that controls the outcome—there is no specific pattern of outcomes. For example, when you create a drawing involving several colors, there is a pattern to the colors producing the design you want. This is a non-random (non-stochastic) process.

The elliptic orbits of most of the planets are examples of outcomes with a definite pattern or design. In this case, the pattern of the elliptic orbit results from the various forces operating on the motion of the bodies, and not from random chance events. Assume that you follow the path of a moving object and that every time the object moves along the path it makes the same turns repeatedly. You would conclude that this pattern of motion results from some non-stochastic process. That is, once you know the pattern you can predict the location of the moving object with great accuracy.

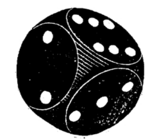
If, on the other hand, the movement of the object resulted from the operation of stochastic processes, a repeating pattern of motion would not occur, and you would not be able to predict with any accuracy the next location of the object as it move down its path. Examples of stochastic processes include, for example: the translational motion of atomic/molecular substances, such as the hydrogen ions in the core of the sun; the outcomes from flipping a coin; etc. Stochastic processes lie behind the design of most games of chance, if they are legitimate.

Stochastic processes have been characterized above as producing no recognizable pattern and having no ability to predict the outcome of future events; yet, some of these outcomes seem to violate these principles. For example, diffusion is the net movement of a substance from an area of greater concentration toward an area of lesser concentration. The end result of diffusion is an even distribution of the substance throughout the volume being considered. This outcome appears to have a definite pattern to it—the movement from an area of greater to an area of lesser concentration. Each time we try this experiment, the same pattern results. Yet, diffusion results from the random translational motion of the atoms/molecules comprising the substance. As Yul Brenner remarked in the musical, *The King & I*, "Is puzzlement!"

PROBABILITY

There is a branch of mathematics that deals with the predicted outcomes of random chance events—it is called probability. The basic rule of probability is: if all outcomes of an event are equally likely (as would be the case for truly random events) the probability that any event will occur is the ratio of the favorable outcomes to possible outcomes. A favorable outcome means the outcome you want to observe, while possible outcomes means any event that can occur. The following puts the definition of probability into algebraic form, where p stands for probability:

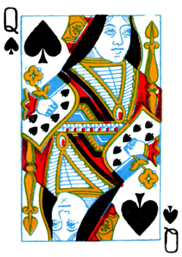
$$P = \frac{\text{\# of favorable outcomes}}{\text{\# of possible outcomes}}$$



Assume that you have a six-sided die (singular for dice) with a different number from one to six on each face of the die. If this is not a "fixed" die, then any one of the faces will have the same chance of facing upward after a given throw. If you want to determine the probability of throwing a six, it would equal the number of favorable outcome (= 1, when the side with the six lands facing upward) divided by the number of possible outcomes (= 6, when any one of the six sides can land facing upward); *i.e.,*

P = 1/6 = (0.17).

Notice that the results of the probability calculations can be expressed as a simple fraction or as a decimal fraction.



Let's take one more example. What is the probability that you will draw the queen of spades from a fully shuffled, legitimate card deck? In this case there is only one favorable outcome (drawing the queen of spades), and there are 52 possible outcomes (drawing any of the 52 cards). Thus, the probability of drawing the queen of spades is:
P = 1/52 = (0.02).

On the other hand, if the task was to draw a queen (regardless of the suit), there now are 4 favorable outcomes (drawing the queen of spades, the queen of hearts, the queen of diamonds, or the queen of clubs constitutes a favorable outcome), and there still are 52 possible outcomes. Thus, the probability of drawing a queen is: **P = 4/52 = (0.08).**

Predicting the outcome of random chance events works for a single occurrence of the event, and it works for predicting the proportion of outcomes when you have many occurrences of the event. For example, the probability that you will withdraw a queen on any given draw is **4/52** or (0.08). If you drew a card 15,000 times from the deck, then you would expect that card to be a queen 15000 x **4/52** or 1154 times.

Notice, if you use the decimal fraction to calculate how many times the queen should appear during 15,000 draws, the predicted number becomes [15000 x 0.08] or 1200. This difference results from having rounded the decimal fraction. Which would you say is the more accurate prediction?

The observed outcomes of random chance events seldom equal the predicted outcomes when only a limited number of observations are made. Probability predictions assume an infinite number of trials of the random chance events, and the more observations of those events that you make, the closer are the observed and predicted outcomes. For example, if you rolled that die just 12 times, it would be quite unusual to observe the six on two of those trials—that is the predicted outcome. On the other hand, if you rolled the die 12,000 times, the chances that you would observe the six on 2,000 of those trials is pretty good. Thus, the probability that the observed and predicted outcomes of random chance events agree increases as the number of trials increases.

STOCHASTIC PROCESSES AND PLANETARY DIVERSITY

The first part of this activity is designed to give you practice in making predictions about the outcomes of stochastic processes and to demonstrate the relationship between predicted and observed outcomes as the number of trials increases.

In the second part of this activity we will explore how the limited number of occurrences of the same stochastic process might result in quite different results, just because we are dealing with a limited number of trials. This part of the activity is meant to model one possible theory for the origin of the planets—that they were formed by stochastic processes from a uniform mixture of the remnants of the solar nebula. It is hard to see how such a theory could explain the formation of planets with different characteristics, since probability reasoning alone would lead us to believe that the elemental composition of all planets should be identical.

However, the formation of the planets represents a limited number of trials of these random chance events—only nine trials! Could the fact that we expect a large deviation between the predicted and observed outcomes of random chance events under these conditions account for the diversity among the planets? This activity is meant to be more of an exploration of this property of stochastic processes, rather than to be a test of a real model for the formation of the planets. While a number of theories do propose that the planets were formed from the solar nebula material that was left over from the formation of the sun, most of them do not propose that the elements within the nebula formed a uniform mixture. What do you think? How do we determine whether or not a patterned outcome results from stochastic or non-stochastic processes? That, certainly, is food for thought!